General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MM04**
Unit Mechanics 4

Thursday 12 June 2008 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

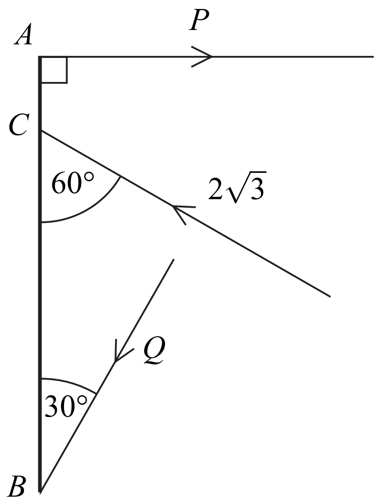
Answer **all** questions.

1 A light rod AB has length 5 metres and the point C on the rod is 1 metre from A. The rod is on a smooth horizontal table and is acted upon by three horizontal forces of magnitude P, Q and $2\sqrt{3}$ newtons.

The force of magnitude P acts at A, at right angles to the rod.

The force of magnitude $2\sqrt{3}$ acts at C, at an angle of 60° to the rod.

The force of magnitude Q acts at B, at an angle of 30° to the rod, as shown in the diagram.



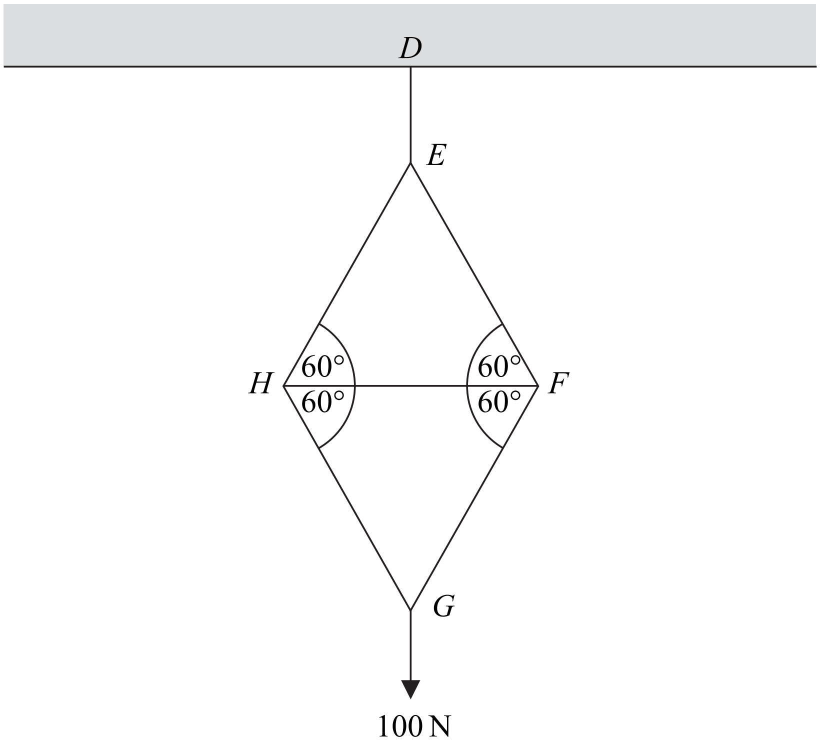
The three forces are equivalent to a couple.

(a) Show that $Q = 2$ and find the value of P. *(5 marks)*

(b) Determine the magnitude of the couple. *(3 marks)*

(c) State the sense of the couple. *(1 mark)*

2 A framework $EFGH$ consists of five identical light rods, EF, EH, FG, GH and FH, which are smoothly jointed at E, F, G and H. Each of the rods EF, EH, FG and GH makes an angle of 60° with the rod FH. The framework is suspended from a fixed point D by a string DE. The rod FH is horizontal, and G is vertically below D. A force of 100 N is applied vertically at G. The system, as shown in the diagram, is in equilibrium.



(a) State the magnitude of the force in the string DE, giving a reason for your answer. *(2 marks)*

(b) Explain why the forces in the rods EF, EH, FG and GH must be of equal magnitude. *(2 marks)*

(c) Find the magnitude of the forces in each of the rods EF, EH, FG and GH. *(2 marks)*

(d) Find the magnitude of the force in the rod FH. *(3 marks)*

(e) State which of the five rods could be replaced by ropes, giving reasons for your answers. *(2 marks)*

Turn over ▶

3 A light rod has its ends at the points $A(2, 3, 5)$ and $B(4, 6, -1)$. A force $\mathbf{F}$ acts at B, where

$$\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$$

(a) Find $\overrightarrow{AB}$. *(1 mark)*

(b) Find the moment of $\mathbf{F}$ about the point A. *(3 marks)*

(c) Show that the magnitude of this moment is $10\sqrt{5}$. *(2 marks)*

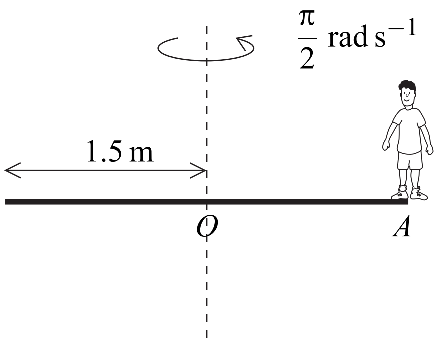
(d) Hence, or otherwise, find the acute angle between $\mathbf{F}$ and the rod, giving your answer to the nearest degree. *(4 marks)*

4 (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}mr^2$. *(5 marks)*

(b) A roundabout in a playground can be modelled as a uniform circular disc of mass 200 kg and radius 1.5 m. The roundabout can rotate freely in a horizontal plane about a vertical axis through its centre O.

The roundabout is rotating at $\frac{\pi}{2}$ radians per second, with Dominic, a child of mass 25 kg, standing at a point A on the edge, as shown in **Figure 1**.

Figure 1

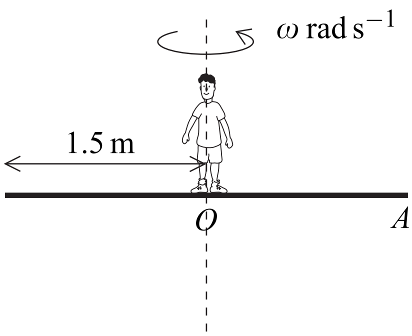


Assume that Dominic can be modelled as a particle.

(i) Show that the moment of inertia of the system about the vertical axis through O shown in **Figure 1** is 281.25 kg m^2. *(3 marks)*

(ii) Dominic then walks to the centre O, as shown in **Figure 2**. The angular speed of the roundabout changes from $\frac{\pi}{2}$ radians per second to ω radians per second.

Figure 2

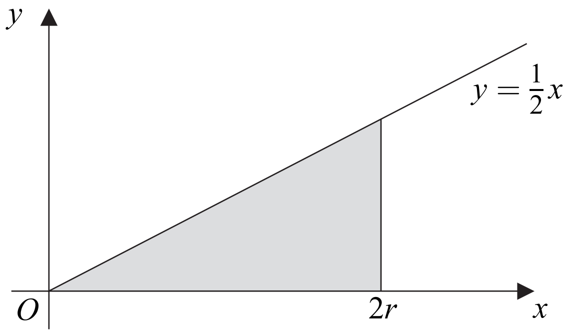


Explain why the total angular momentum of the system remains constant as Dominic walks from A to O. *(1 mark)*

(iii) Find the value of ω. *(4 marks)*

Turn over ▶

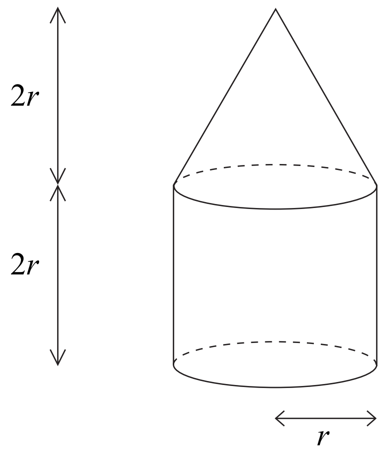
5 The region bounded by the line $y = \frac{1}{2}x$, the x-axis and the line $x = 2r$ is shown in the diagram.



This region is rotated about the x-axis to form a uniform solid cone of height $2r$ and radius r.

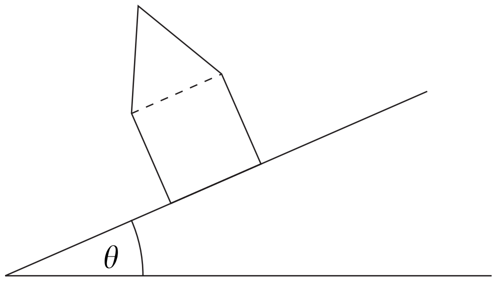
(a) Show, using integration, that the centre of mass of the cone is at a distance of $\frac{3r}{2}$ from the origin. *(5 marks)*

(b) A rocket consists of two parts. The lower part of the rocket may be modelled as a uniform solid cylinder with radius r, height $2r$ and density ρ. The upper part of the rocket may be modelled as a uniform solid cone of radius r, height $2r$ and density $k\rho$, as shown in the diagram.



(i) Show that the centre of mass of the rocket is at a distance of $\left(\frac{6+5k}{6+2k}\right)r$ from the base of the rocket. *(5 marks)*

(ii) The rocket is now placed on a rough plane, which is inclined at an angle of θ to the horizontal, where $\tan\theta = \frac{2}{3}$.

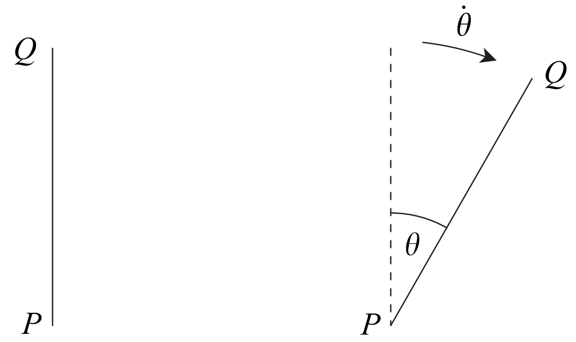


Given that the rocket does **not** slide and is just on the point of toppling, find the value of k. *(5 marks)*

Turn over for the next question

6 A uniform rod PQ, of mass m and length $6a$, is free to rotate in a vertical plane about a fixed horizontal axis through P. Initially, the rod is at rest with Q vertically above P.

The rod is slightly disturbed from its initial position. In the subsequent motion, it makes an angle θ with the upward vertical at time t.



Initial position During motion

(a) (i) Show that the moment of inertia of the rod about the axis through P is $12ma^2$. *(1 mark)*

(ii) Show that $\dot{\theta}^2 = \dfrac{g}{2a}(1 - \cos\theta)$. *(4 marks)*

(iii) Hence, or otherwise, determine an expression for $\ddot{\theta}$ in terms of a, g and θ. *(2 marks)*

(b) Find, in terms of m, g and θ, the force at P which the axis exerts on the rod:

(i) in the direction PQ; *(4 marks)*

(ii) perpendicular to PQ. *(3 marks)*

(c) Determine the magnitude of the force exerted by the axis on the rod when Q is vertically below P. *(3 marks)*

END OF QUESTIONS